

CRITERIAL GENERALIZATION OF EXPERIMENTAL RESULTS FOR THE THERMAL  
AND ELECTRICAL CHARACTERISTICS OF A GLOW DISCHARGE

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Criterial equations are obtained for the generalization of the electrical and thermal characteristics of a glow discharge. The experimental results are given in the form of generalized graphs and empirical formulas.

The well-known Paschen law  $U_B = f(P \cdot l)$ , the result of applying the methods of similarity and dimensionality theory to an electrical discharge in motionless gas, has played a large role in the generalization of experimental data on breakdown [1]. Recently, these methods came to be used in generalizing the characteristics of discharges in gas flows. The first successful step in the criterial generalization of U-I characteristics of arcs with eddy stabilization was made in [2], where their generalized U-I characteristic was written in the form of a dependence of U on the dimensionless factor  $c_1 I^2 / Gd$ , where  $c_1$  is a dimensional coefficient. In [2], the factor  $c_1 I^2 / Gd$  was obtained from the energy-conservation equation by reducing it to dimensionless form. Subsequently, criterial generalization was also extended to other electrical and thermal characteristics of a stabilized arc in plasmotrons [3, 4]. Considerable experimental material on electric arcs has now been accumulated, and generalized using criterial equations in the form of empirical formulas [5]. These formulas are widely used in engineering calculations of plasmotron arcs.

In recent years, in connection with the great promise of nonequilibrium plasma for use in various areas of economic activity, there have been intensive attempts to create plasmotrons with a glow discharge (PGD). In view of the extreme complexity of the processes in PGD, experiment plays the main role in their investigation, and correspondingly engineering calculations and projections are based on experimental data. Therefore, there is an urgent need to develop methods of criterial generalization of the data and of plotting generalized characteristics of a glow discharge in gas flows. The aim of the present work is to analyze the similarity groups, obtain criterial equations, and plot generalized characteristics of a glow discharge in a cylindrical channel with a gas flow, serving as a PGD discharge chamber.

A diagram of a discharge chamber (DC) with a longitudinal gas flow is shown in Fig. 1. It is the simplest to analyze and at the same time very widely used. The basic components are a copper cathode 1 and anode 2 and a quartz interelectrode insert (IEI) 3. Gas passing through the positive column 4 of the discharge is subject to ionization, nonequilibrium heating, and dissociation, and various chemical reactions occur in it. In a glow discharge, the electron-gas temperature is usually in the tens of thousands of degrees, whereas the translational temperature of the heavy particles does not exceed 1000°K. The disequilibrium significantly complicates the analysis of processes in PGD and glow-discharge theory.

Assume that there is no external magnetic field and the discharge is independent. The parameters which are important for processes in DC are  $m_a, m_e, m_i$  ( $i = 1, 2, \dots, n$ ),  $U_j$  ( $j = 1, 2, \dots, v$ ),  $Q_k$  ( $k = 1, 2, \dots, s$ ),  $T_b, T_R, I, e, U_a, U_c, \varphi$ , the characteristic values  $P, V_z, V_r, V_\varphi$  and the quantities  $d, d_a, l$  shown in Fig. 1. If the dimensionalities of temperature and current are taken as the primary dimensionalities, the Boltzmann constant  $k$  and the electrical constant  $\epsilon_0$  must be included in the system of determining parameters. Thus the processes are determined by  $(n + s + v + 18)$  independent quantities, whose dimensionalities may be written in terms of five primary dimensionalities. In accordance with the  $\pi$  theorem of dimensionality theory,  $(n + s + v + 13)$  independent dimensionless factorial combinations may be written on the basis of these quantities [6]. Taking account of the results of [2, 3, 4, 7], the simplest combinations will be taken

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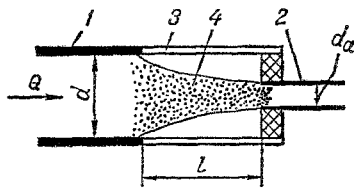


Fig. 1. Diagram of the discharge chamber.

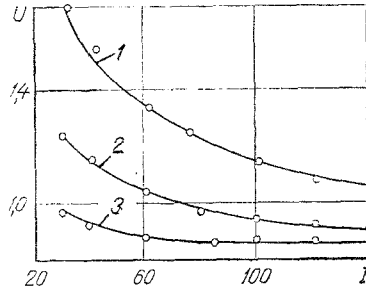


Fig. 2. U-I characteristics of a glow discharge in air flow: 1)  $P = 4.53$  kPa,  $F = 2.6 \cdot 10^{-4}$  kg/sec,  $d = 2.3 \cdot 10^{-2}$  m,  $L = 4.4 \cdot 10^{-2}$  m; 2) 3.73,  $1.1 \cdot 10^{-4}$ ,  $2.3 \cdot 10^{-2}$ , and  $4.4 \cdot 10^{-2}$ ; 3) 7.73,  $4.3 \cdot 10^{-5}$ ,  $0.9 \cdot 10^{-2}$ , and  $1.7 \cdot 10^{-2}$ . I, mA; U, kV.

$$\frac{m_e}{m_a}, \frac{m_i}{m_a} (i = 1, 2, \dots, n), \frac{Q_k}{Q_1} (k = 2, 3, \dots, s),$$

$$\frac{U_j}{U_1} (j = 2, 3, \dots, v),$$

$$\frac{U_a}{U_1}, \frac{U_c}{U_1}, \frac{\varphi}{eU_1}, \frac{eU_1}{kT_b}, \frac{V_\varphi}{V_z}, \frac{V_r}{V_z}, \frac{d_a}{d}, \frac{l}{d}, \frac{T_R}{T_b}, \frac{\lambda}{d},$$

$$\frac{m_a I}{eG}, \frac{dPQ_1 m_a^{0.5} V_z}{(kT_b)^{1.5}}, \frac{e^2 N^{1/3}}{\epsilon_0 kT_b}, \frac{Q_1^{1.5} P}{kT_b},$$
(1)

where

$$G = \frac{\pi d^2 \rho V_z}{4}, \quad \lambda = \frac{kT_b}{PQ_1}, \quad \rho = \frac{Pm_a}{kT_b}, \quad N = \frac{P}{kT_b}.$$

All the definable dimensionless parameters characterizing the processes in DC are functions of the similarity groups in Eq. (1). The numbers  $m_e/m_a$ ,  $m_i/m_a$ ,  $Q_k/Q_1$ ,  $U_j/U_1$  for different gases are different. Hence, in the general case, glow discharges in two different gases are nonsimilar. Therefore, discharges in a single gas will be considered below. In this case, the given numbers will be constants, and may be excluded from consideration. If the electrodes of comparable DC are made from the same materials, then  $\varphi/eU_1$  is also a constant. Usually  $T_b$  and  $T_R$  change very slightly, so that it is possible to write

$$eU_1/kT_b = \text{const}, \quad T_R/T_b = \text{const}.$$

The number  $(e^2 N^{1/3})/(\epsilon_0 kT_b)$  is proportional to the ratio between the potential energy of the interaction of unlike charged particles and their thermal energy. In glow-discharge conditions, the gas is ideal, and the given number, characterizing the deviation from ideality, may be eliminated from consideration. In the first approximation,  $U_a$  and  $U_c$  may be regarded as constant. In addition, it is assumed that, for comparable DC,  $V_\varphi/V_z$  and  $V_r/V_z$  are, respectively, the same. The numbers  $Q_1^{1.5} P/kT_b$  are proportional to the ratio between the sum of molecular volumes of the gas and the volume which the gas molecules occupy. It may be discarded as insignificant. In these conditions, the variables that remain are

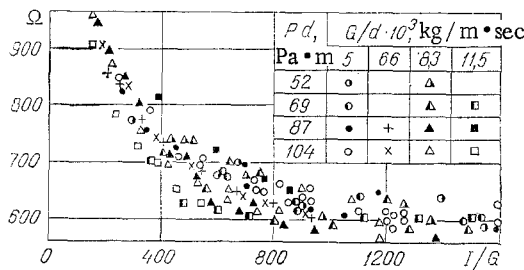


Fig. 3. Generalized U-I characteristic of a glow discharge in an air flow. I/G, A·sec/kg.

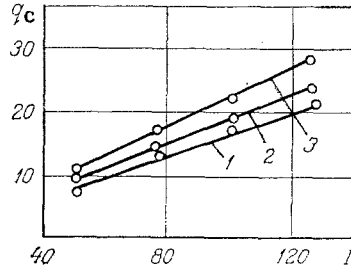


Fig. 4. Heat loss through the cathode when  $G = 1.1 \cdot 10^4$  kg/sec;  $d = 2.3 \cdot 10^{-2}$  m;  $l = 4.4 \cdot 10^{-2}$  m; 1)  $P = 4$  kPa; 2) 4.53; 3) 5.33.  $q_c$ , W.

$$Kn = \frac{\lambda}{d}, \quad K_1 = \frac{m_a l}{eG}, \quad K_2 = \frac{d \rho V_z Q_1}{(m_a k T_b)^{0.5}}, \quad K_3 = \frac{d_a}{d}, \quad K_4 = \frac{l}{d}.$$

Here  $Kn$  is the Knudsen number, while  $K_2$  is proportional to the Reynolds number. Writing  $Kn$  and  $K_2$  in the form

$$Kn = \frac{c_n}{Pd}, \quad K_2 = \frac{c_2 G}{d}, \quad c_n = \frac{k T_b}{Q_1}, \quad c_2 = \frac{4 Q_1}{\pi (m_a k T_b)^{0.5}},$$

the criterial equation for the integral dimensionless quantity  $\Psi_1$  is written in the form

$$\Psi_1 = f_1(Kn, K_1, K_2, K_3, K_4). \quad (2)$$

The quantity  $\Psi_1$  may be the dimensionless discharge potential  $U/U_1$  and the heat loss  $q/IU$  and other dimensionless parameters.

The distribution of the determinable parameters along DC is described by the equation

$$\Psi_2 = f_2 \left( Kn, K_1, K_2, K_3, K_4, \frac{z}{d} \right). \quad (3)$$

The quantity  $\Psi_2$  may be the dimensionless electric field strength, heat flux, etc. In writing Eqs. (2) and (3), the influence of some groups has been neglected. However, combinations of these groups with others may play a definite role in the description of the given processes. The role of such overlap effects and the correctness of the given assumptions must be studied in more detail in the future, invoking experimental data.

In the conditions corresponding to the given assumptions,  $c_n$ ,  $m_a/e$ , and  $c_2$  are constant. Therefore, in order to simplify the expressions, they may be neglected, retaining only the variable parts  $I/G$ ,  $Pd$ , and  $G/d$  of the corresponding similarity groups. Thus, the simplified equations are written in the form

$$\Psi_1 = f_1 \left( \frac{I}{G}, Pd, \frac{G}{d}, \frac{d_a}{d}, \frac{l}{d} \right), \quad (4)$$

$$\Psi_2 = f_2 \left( \frac{I}{G}, Pd, \frac{G}{d}, \frac{d_a}{d}, \frac{l}{d}, \frac{z}{d} \right). \quad (5)$$

Typical U-I characteristics for several values of P, G, d, and l are shown in Fig. 2, from which it is evident that the discharge potential grows with increase in Pd. Analysis of the experimental data shows stratification of the volt-ampere characteristics (VAC) with respect to identical Pd and G/d. The VAC of the discharge shown in Fig. 2 are declining. An analogous declining VAC was given in [8] for a glow discharge in a cylindrical sealed tube at pressures of 0.1-1.0 kPa. In [9], VAC-curve variation of this kind was attributed to the influence of gas heating. The results of experiments to determine the distribution of the potential for parameter variation in the ranges P = 2.7 - 11.3 kPa, G = 4.5·10<sup>-5</sup>-2.6·10<sup>-4</sup> kg·sec<sup>-1</sup>, d = 0.9·10<sup>-2</sup> - 2.3·10<sup>-2</sup> m, and I = 30 - 150 mA give a value of 250-300 V for the cathode drop. Thus, the given discharge is a glow discharge.

For a glow-discharge potential, Eq. (4) is written in the form

$$\frac{U}{U_1} = f_1\left(\frac{I}{G}, Pd, \frac{G}{d}, \frac{d_a}{d}, \frac{l}{d}\right). \quad (6)$$

Here U<sub>1</sub> is a constant. The discharge chambers investigated are geometrically similar, i.e., they had the same values of d<sub>a</sub>/d and l/d. Taking account of these factors, the generalized VAC may be written in the form

$$U = U_1\left(\frac{I}{G}, Pd, \frac{G}{d}\right). \quad (7)$$

In Fig. 3, the generalized VAC is shown in the form of a dependence of the quantity Ω = U(G/d)<sup>-α</sup> (Pd)<sup>-β</sup> on I/G. With an error of ±8%, it is described by the empirical formula

$$U = 2150\left(\frac{G}{d}\right)^{0.1} (Pd)^{0.2} \left(\frac{I}{G}\right)^{-0.18}. \quad (8)$$

The formula obtained covers the parameter ranges: P = 2.7-11.3 kPa, G = 4.5·10<sup>-5</sup>-2.6·10<sup>-4</sup> kg/sec, d = 0.9·10<sup>-2</sup>-2.3·10<sup>-2</sup> m, and I = 30-150 mA.

Numerous experimental data show that the U-I characteristics in the experimental discharge chambers are repeatable with an accuracy of ±5% in the same external conditions, and therefore the accuracy of description of the U-I characteristics by Eq. (8) may be regarded as satisfactory.

The dependence of the heat loss through the cathode on the discharge current in the pressure range 4.0-5.3 kPa is shown in Fig. 4. With increase in pressure in DC, q<sub>c</sub> rises. Analysis of the experimental results gives the empirical dependence

$$q_c = 0.8 IP^{0.65}, \quad (9)$$

describing the experimental data with an error of ±4%. Analysis of the experimental data shows that, in the range G = 4.3·10<sup>-5</sup>-1.1·10<sup>-4</sup> kg/sec, q<sub>c</sub> does not depend on the gas flow rate. The heat loss through the interelectrode insert is described, with an error of ±5%, by the empirical formula

$$q_M = 4650 I \cdot G^{0.28}. \quad (10)$$

In the future, when experimental material has been collected, it will be necessary to refine Eqs. (9) and (10) and to pass to more general expressions of the form

$$\frac{q_c}{IU} = f_3\left(\frac{I}{G}, Pd, \frac{G}{d}\right), \quad \frac{q_M}{IU} = f_4\left(\frac{I}{G}, Pd, \frac{G}{d}\right).$$

Thus, criterial equations have been written to generalize experimental data, generalized graphs have been plotted, and empirical formulas have been obtained for the U-I characteristics of a glow discharge, and the heat loss through the cathodes and the IEI of the discharge chamber.

Further experimental investigations in a wider range of variation of the determining parameters are required to refine the empirical formulas.

#### NOTATION

U<sub>B</sub>, breakdown potential; U, discharge potential; U<sub>C</sub>, U<sub>A</sub>, cathode and anode potential; drop; U<sub>i</sub>, ionization, dissociation, and excitation potentials; φ, electron work function; I, discharge current; ε<sub>0</sub>, electrical constant; e, electron charge; Q<sub>k</sub>, cross section of collision and various elementary processes; m<sub>a</sub>, mass of initial gas molecule; m<sub>e</sub>, electron mass;

$m_i$ , mass of newly formed gas molecule;  $G$ , second mass gas flow rate;  $V_z, v_r, v_\phi$ , projections of gas velocity vector;  $P$ , gas pressure;  $k$ , Boltzmann constant;  $\rho$ , density;  $\lambda$ , free path length;  $T_b$ , gas temperature at the discharge-chamber inlet;  $T_R$ , temperature of inner channel wall;  $q_C, q_M$ , loss of heat through cathode and interelectrode insert;  $d$ , discharge-chamber diameter;  $d_a$ , anode diameter;  $l$ , distance between electrodes;  $c_1, c_2, c_n$ , constant dimensional coefficients.

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#### SPECIFIC HEAT OF LIQUID CESIUM AT TEMPERATURES UP TO 2000°K AND PRESSURES UP TO 12 MPa

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We present and discuss measured values of the specific heat and resistivity of liquid cesium in the temperature range 1200-1950°K at pressures up to 12 MPa.

Considerable progress has been made in the last decade in the study of the thermodynamic and electrical properties of liquid alkali metals and mercury over the whole existence domain of the liquid phase and also in the supercritical region. The results obtained are of interest both from the point of view of understanding the laws of behavior of the thermodynamic parameters of liquid metals over a wide range of states, and also from the point of view of clarifying the nature of the metal-dielectric transition in liquid metals. The aim of the present study is to supplement existing knowledge by data on the caloric properties of the alkali metals (e.g., cesium) in the existence domain of the liquid phase. The experimental results in [1] are discussed here together with new data obtained recently.

The measurements at high temperatures under pressure were made by a method based on the periodic heating of the sample in a high-pressure chamber by an electric current. The measuring cell (sample) was a thin molybdenum tube 150 mm long with a 5.2-mm inside diameter and a 0.15-0.2-mm wall thickness filled with the liquid metal and pressurized by a bellows welded onto the upper end of the cell. The high-pressure chamber was provided with leucosaphire-optical windows which permitted the determination of the reference temperature with an OP-70 optical pyrometer, and the recording of temperature oscillations of the sample surface by a photoelectric described in more detail in [2, 3].

An important feature of the method used is the fact that the sample under study was in a gaseous atmosphere. This necessitated estimating the effect of heat transfer on the specific heat measurements. We also took account of the fact that the sample was a two-layer system consisting of a liquid metal core in a molybdenum shell, and that the components of the system had very different properties. It was necessary to choose and produce experimental conditions to minimize the effect of these factors. To this end we investigated the system of two

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